OPTIMISTIC & PESSIMISTIC TRADING IN FINANCIAL MARKETS

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Abstract

There is an abundant literature in finance on overconfidence, however there exists a different psychological trait well known to financial practitioners and psychologists [see Hilton at al. (2004)] which is optimism. This trait has received little attention. Our paper analyses the consequences of optimism and pessimism on financial markets. We develop a general model of optimism/pessimism where M unrealistic informed traders and N realistic informed traders trade a risky asset with a competitive market maker. Unrealistic traders can (i) misperceive the expected returns of the risky asset (scenario 1) or (ii) in addition to the previous can make a judgmental error on both the volatility of the asset returns and the variance of the noise in his/her private signal (scenario 2). We show, in scenario 1, that optimistic traders purchase more or sell smaller quantities whereas pessimistic traders sell more or purchase smaller quantities than if they were realistic. As market makers correctly anticipate that, we obtain that (i) the price level and the market depth are equal to the ones predicted by a standard model a la Kyle (1985) with $M + N$ realistic traders and (ii) unrealistic and realistic traders obtain the same expected profit. In scenario 2, we show that (i) unrealistic traders may earn negative, higher or lower expected profit than realistic traders, (ii) the expected profit of the realistic and the unrealistic trader is a non-monotonic function of the two degrees of error, and (iii) market depth is also a non-monotonic function of the error on the volatility of asset as well as the error on the variance of the noise. All of the above results are derived when market makers are realistic. We show that the results are not altered if market makers are themselves optimistic or pessimistic. The expected profit for the unrealistic market makers can either be positive or negative.

1 Introduction

Economic and financial theory have widely used the assumption that agents behave rationally. Such an assumption has led to the failure of explaining some properties observed in financial markets like (*i*) the low responsiveness or sometimes high responsiveness of the price to new information [Ritter (1991) and Womack (1996)], (*ii*) the excessive volume traded [Dow and Gorton (1997)], (*iii*) underreaction or overreaction of market participants [Debondt and Thaler (1985)], and (*iv*) the excessive volatility observed in financial markets [Shiller (1981, 1989)]. In order to explain these properties, financial economists have departed from the rationality assumption and have instead assumed that investors may have some psychological traits which would lead them to behave irrationally.

There is a large body of evidence in the psychology literature that people, with good mental health, do not have accurate perception of themselves and their surrounding world. People's perceptions have a tendency to be positively biased, i.e. people hold "positive illusions". "Positive illusions" have been widely documented in psychology [Taylor and Brown (1988, 1994), Langer (1974) and Weinstein (1980), to name but a few]. Taylor and Brown (1988, 1994) analyze the "better than average" effect, Langer (1974) focuses on the illusion of control i.e. the individuals' tendency to overestimate the control they have over outcomes, whereas Weinstein (1980) looks at unrealistic optimism. Unrealistic optimism is defined as the people's tendency to systematically overestimate the probability that good things will happen to them and, at the same time, to underestimate the probability that bad things will happen.[1] One way psychologists test the presence of that trait is to ask their subjects how they behave in bad times. If in bad times they expect the best, they can be considered as optimistic.

Financial pratictioners are well aware of the existence of such a trait in markets. Indeed the terminology used in order to qualify the market proves that fact: they interchangeably use optimistic market for bullish market and pessimistic market for bearish market. Some financial institutions have tried to quantify it. In October 1996, the Union des Banques Suisses together with Gallup Organization have launched the *Index of Investor Optimism.* This index measures the level of optimism in the American market.[2]

Although the existence of such a trait is established, it has received little attention by financial economists. Our aim is to fill up that gap. Indeed, our study follows directly the idea that some investors display a psychological trait such as "positive illusions/negative illusions". We call the former (latter) investors optimists (pessimists). We model this trait as being made up by two independent parts: (*i*) pure optimism/pessimism: misperception of prior information (expectation and variance) and, (*ii*) pure overconfidence/underconfidence: misperception of the variance of the noise in the private signal. Few recent papers analyze this trait, however all these studies do not look at the impact of the presence of unrealistic traders in financial markets. Most of these studies are corporate finance oriented. Bénabou and Tirole (2002) look at the value of self-confidence for rational agents and at their behavior used to enhance it. "Positive thinking" is found to improve welfare despite the fact that it can be self-defeating. In Brocas and Carrillo (2004), optimism about the chance of success of a project may lead entrepreneurs to invest in that project without gathering further information. This entrepreneurial

[1] Harris and Middleton (1994), and Hoorens (2001) look at the "comparative optimism". These two papers focus on the illusion of control and optimistim about health: people think on being less at risk but no more in control than others.

[2] A detailed methodology used to compute that index can be found at www.ubs.com/investoroptimism. The monthly level of the index is also given from its launch date up to now.

optimism leads to excessive investment. De Mezza and Southey (1996) find that entrepreneurs self select from the part of the population displaying an optimistic bias. Manove and Padilla (1999) look at the screening problem faced by bankers in order to separate optimistic entrepreneurs from realistic ones. Optimistic entrepreneurs have perceptions biased by wishful thinking. They show that, because of the existence of optimistic entrepreneurs, competitive banks may not be sufficiently conservative in their lendings. In Manove (2000), some entrepreneurs are unrealistically optimistic about their firms productivity. He shows that unrealistic entrepreneurs may earn more than realistic entrepreneurs and may even drive out of business all realistic entrepreneurs. This result is echoed by Heifetz and Spiegel (2004) in a different framework. In a game where agents meet per pair and then interact, they show that agents displaying optimism or pessimism will not disappear in the long run but will takeover the entire population. In their paper, optimistic (pessimistic) agents over- (under-) estimates the impact of their actions. Our study differ from the previous studies on optimism as none of the above works look at the impact on financial markets of having optimistic (pessimistic) traders. Cornelli et al. (2005) is the only other work, we are aware of, looking at the possibility for a subset of investors to be optimistic or pessimistic. They establish the existence of unrealistic traders possibly optimistic or pessimistic in the grey market (pre-IPO market). In our paper, we assume the existence of a subset of informed investors being optimistic and/or pessimistic and derive the equilibrium and its properties under that assumption.

The introduction of a psychological bias or trait at the investors' level is not new in finance, a proof being the abundant literature assuming that investors are overconfident, i.e. that investors tend to overestimate the probability that their judgments are correct.[3] Hilton et al. (2004) show that overconfidence or "miscalibration" and "positive illusions" differ even if those two traits share some characteristics (in both cases the agent over-values his/her ability). In order to model the overconfident behavior, it is assumed that investors overestimate the precision of their private information. Most of that literature predicts that overconfident investors trade to their disadvantage. In other words, overconfident investors get lower expected profit than their rational counterpart [Odean (1998b), Gervais and Odean (1999), Caballé and Sákovics (2003), Biais et al. (2004) among others]. However, Kyle and Wang (1997) and Benos (1998) find that overconfident traders may earn larger expected profit than rational ones. Moreover, a common finding to all these papers except Caballé and Sákovics (2003) is that trading volume, price volatility as well as price efficiency increase with the level of overconfidence.

We study a model where a market maker and several traders exchange a risky asset that is normally distributed with mean 0 and variance σ_v^2. That model is based on the seminal paper by Kyle (1985) where the traders can be either be uninformed (noise traders) or informed. In such a framework, unrealism can be modelled in two different ways. First, unrealistic traders can be unrealistic about the returns of the risky asset and therefore believe that the expectation is a rather than zero. An optimist (pessimist) believe that a is positive (negative), as a increases in absolute terms the trader is more optimistic/pessimistic. This is studied in a first setting. Second, unrealistic traders can be unrealistic about the returns of the risky asset, as in the first setting, and about the volatility of the returns of the asset as well as the variance of the noise in his private information (over- or underconfidence). This is analyzed in a second setting. We model the misperceptions of the variances as two parameters: κ_1 (optimistic/pessimistic component) and κ_2 (overconfident/underconfident component) which altogether characterize the degree of misperception of the two variances. The smaller the κ parameter, the more un-

[3] See Hilton et al. (2001) and Biais et al. (2004).

realistic is the trader. An optimistic (pessimistic) trader under-scales (over-scales) the volatility of the risky asset σ_v^2 by a parameter κ_1 and under-scales (over-scales) the precision of his own signal σ_ε^2 by a parameter κ_2.[4] In both settings, traders are strategic, i.e., when computing their order they take into account the impact of the order onto the price.
In the first setting (misperception of the returns only), we find that, compared to the case where only realistic traders are present (all realistic case), optimistic (pessimistic) traders purchase (sell) larger quantities or sell (purchase) smaller quantities although without altering their information revelation. This additive part in the market order is proportional to the misperception a. Even though, the aggregate order flow faced by the market maker is different from the one in the "all realistic case", the price and the market depth are equal for the two cases.[5] Finally, the expected profits of the realistic and the unrealistic traders are identical. In that market it is not costly to be unrealistic. In the second setting (misperception of the returns and of the two variances), we look at two different situations. In the first one, a realistic market maker trades with realistic traders and with only one type of unrealistic traders. The second situation studies the case where the market maker is herself optimistic or pessimistic. The market maker's unrealism is characterized by the misperception of the returns of the asset and the misperception of its volatility. Given that the market maker does not receive any private information, she does not display any overconfidence or underconfidence trait.[6] In both instances, unrealistic and realistic traders, as a response to the unrealistic traders' behavior, alter their information responsiveness (trading intensities). According to intuition, we find that, compared to a situation where all traders are realistic, realistic traders reduce (increase) their trading intensity when unrealistic traders over- (under-) trade. However, we show that there exists a situation where both types of traders over-trade. This happens to be the case when the misperception of the volatility is smaller, however not to small, than the misperception of the noise in the signal ($\kappa_1 > \kappa_2$). In that case, the unrealistic traders' order impact is high, as their trading intensity is high, and realistic traders find profitable to over-trade. The trading behavior has an effect on market depth. it can be higher or lower than in the "all realistic case". Importantly, we show that, for both a realistic and an unrealistic market maker a market breakdown occurs if both types of traders over-trade excessively. The market breakdown is more likely to occur with an optimistic market maker than with a pessimistic one. Indeed, an optimistic (pessimistic) market maker sets a higher (smaller) market depth than a realistic one. An optimistic (pessimistic) market maker thinks that prior information is more (less) precise than it is and therefore believes that the informed private information is less (more) substantial than it is in reality. As a consequence, she adjusts her price less (more) aggressively. This is done by increasing (decreasing) market depth. As a response to the increased (decreased) market depth , both types of traders increase (decrease) their trading intensity. As they increase (decrease) their trading intensity, a market breakdown is more (less) likely to occur.

We finally show that, when compared to the "all realistic case" unrealistic traders over-trade and realistic traders under-trade in order to reduce the impact of their order onto the price, as it then reduces the impact of the unrealistic market orders

[4]In Odean (1998b) those two parameters define the level of overconfidence. Nevertheless, the author does not study the impact of the variations of those parameters on the traders and Odean (1998b) assumes that κ_1 is greater than one whereas we assume that for optimistic traders it is lower than one.

[5]The price and market depth are equal to the one predicted by a standard model a la Kyle (1985).

[6]Assuming that the market maker is unrealistic enables us to study the impact of unrealistic liquidity suppliers in standard demand and supply framework.

onto the price, unrealistic traders earn on average more than their realistic counterpart. In all remaining cases, the unrealistic traders earn on average less than the realistic traders with negative expected profit when both types over-trade. We, also, show that an unrealistic market maker can obtain non-zero expected profit.

In other models like Benos (1998) or Daniel, Hirshleifer and Subrahmanyam (2000) overconfidence is defined by κ_2, only. By allowing these two parameters to characterize the unrealistic behavior, our findings are more general than the ones obtained in Benos (1998), Kyle and Wang (1997), Daniel, Hirshleifer and Subrahmanyam (2000) to name but a few.[7] Since overconfident traders do not misperceive the expectation of the risky asset, the first effect is not present with overconfident traders. The effect of the parameter defining the over- or under-confidence is known, however the combination of the two misperceptions of the variances is not present when traders are only overconfident. Some of our results are qualitatively and quantitatively different from the ones obtained with overconfident traders. Indeed, the price function displays properties different than the ones obtained in the overconfident case. The presence of optimistic traders has an effect on both the intercept and the slope of the price function. The first effect is not present in any of the studies with overconfident traders. The second effect is present with overconfident traders, however we find that market liquidity, depending on the values of the parameters, can be greater or smaller than the market liquidity with realistic traders only. Benos (1998), the closest paper to ours, finds that the market liquidity is increased if overconfident traders are present.

The paper unfolds as follows. In the next section, the general model is presented with the definition of an equilibrium in our model. In section 3, the model is solved for the additive misperception (misperception of the returns) alone as well as for the case where the additive and the multiplicative (misperception of the variances) misperceptions are combined for the case when the market maker is realistic. In section 4, we derive the equilibrium under the assumption that the market maker is herself optimistic or pessimistic and trades with only one type of unrealistic traders. The last section summarizes our results and concludes. All proofs are gathered in the appendix.

2 Model

We study a financial market where a market maker and several traders exchange a risky asset whose future value $\tilde{v}$ follows a gaussian distribution with zero mean and variance σ_v^2. Traders participating in that market can either be informed or uninformed. The uninformed traders are the so-called noise traders and submit a market order which is the realization of a normally random variable $\widetilde{u}$ with zero mean and variance σ_u^2. The informed traders are risk neutral and can be one of two types: realistic or unrealistic. N traders are realistic whereas M are unrealistic. Both type of traders observe a noisy signal of the future value of the risky asset

$$\tilde{s}_k = \tilde{v} + \tilde{\varepsilon}_k, \text{ with } \tilde{\varepsilon}_k \to N\left(0, \sigma_\varepsilon^2\right) \quad \forall k = 1, ..., N, N+1, ..., N+M.$$

These two types of traders differ in the beliefs they have about the expectation of the risky asset value or the expectation of prior information. Realistic traders, correctly, believe that the expectation is zero. However, unrealistic traders believe that it is a. That expectation is positive (negative) if their are optimistic (pessimistic). The term unrealistic trader refers to a trader who uses the wrong distribution for the

[7] All these authors implicitly assume that $\kappa_1 = 1$. Benos (1998) studies the extreme case where overconfident traders perceive their private information as being non noisy, that is $\kappa_1 = 1$ and $\kappa_2 = 0$.

asset return. However that trader rationally anticipates the behavior of both the market maker and the remaining informed traders.

The strategy of each realistic trader i is a Lesbegue measurable function, $X_i : \Re \rightarrow \Re$, such that $\tilde{x}_i = X_i(\tilde{s}_i)$ for $i = 1, ..., N$. The strategy of the unrealistic is identically defined: $X_j : \Re \rightarrow \Re$ such that $\tilde{x}_j = X_j(\tilde{s}_j)$ for $j = 1, ..., M$.

Finally, the market maker is risk neutral and behaves competitively. She observes the aggregate order flow $\tilde{y} = \sum_{i=1}^{N} \tilde{x}_i + \sum_{j=1}^{M} \tilde{x}_j + \tilde{u}$ before setting the price $\tilde{p}$. Let $P : \Re \rightarrow \Re$ denote a measurable function such that $\tilde{p} = P(\tilde{y})$.

The trading protocol is identical to Kyle (1985).

We now give the definition of an equilibrium for our model.

Definition $(X_1^r, ..., X_N^r, X_1^{un}, ..., X_M^{un}, P) \in L^{N+M+1}$ *is an equilibrium if the price set by the market marker is such that*

$$\tilde{p} = E\left[\tilde{v} \middle| \tilde{y}\right],$$

and for the market orders submitted by each trader, given that price, the market orders should maximize the traders' expected profit conditional on the information received

$$X_i^r \in \arg\max_{x_i^r \in \Re} E\left[(\tilde{v} - P(\tilde{y}))\, x_i^r \middle| s = s_i\right] \quad \forall i = 1, ..., N,$$

and

$$X_j^{un} \in \arg\max_{x_j^{un} \in \Re} E_{un}\left[(\tilde{v} - P(\tilde{y}))\, x_j^{un} \middle| s = s_j\right] \quad \forall j = 1, ..., M.$$

The operator E_{un} denotes the fact that the expectation for unrealistic traders, is computed given their beliefs about the risky asset's expectation.

It should be pointed out that all traders know their types. Moreover, all agents know the number of realistic and unrealistic traders as well as if unrealistic traders are optimistic or pessimistic. Traders behave strategically meaning that they take into account the impact of their orders onto the price.

First, we solve the model for the case where the market maker is realistic, i.e. she uses the correct distribution of the asset. Second, we look at the case where she is either optimistic or pessimistic.

3 Realistic Market Makers

3.1 Additive Misperception

In that subsection we derive the equilibrium where the unrealistic traders have incorrect beliefs about the expectation of the risky asset value. Let us define $\tau = \frac{\sigma_\varepsilon^2}{\sigma_v^2}$.

Proposition 1 *There exists an equilibrium of the following form*

$$\begin{aligned} x_i^r &= \alpha^r + \beta^r s_i, \quad \forall i = 1, ..., N, \\ x_j^{un} &= \alpha^{un} + \beta^{un} s_j, \quad \forall j = 1, ..., M, \end{aligned}$$

$$p = \mu + \lambda y = \mu + \lambda \left(\sum_{i=1}^{N} x_i^r + \sum_{j=1}^{M} x_j^{un} + u \right).$$

The coefficients for the market orders are such that

$$\alpha^{un} = \frac{2a\sigma_u\tau}{\sigma_v\sqrt{(M+N)(1+\tau)}}, \alpha^r = 0,$$
$$\beta^{un} = \beta^r = \frac{\sigma_u}{\sigma_v\sqrt{(M+N)(1+\tau)}}.$$

The price schedule is given by

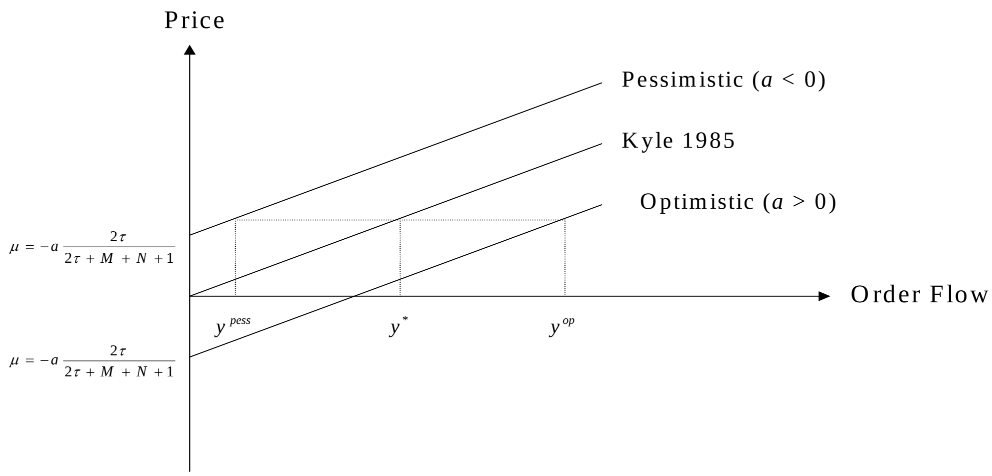
$$\mu = -a\frac{2\tau}{2\tau+M+N+1},$$
$$\lambda = \frac{\sigma_v\sqrt{(M+N)(1+\tau)}}{\sigma_u(2\tau+M+N+1)}.$$

Proof. See Appendix. ■

In the following discussion we only look at the case where optimistic traders are present as the pessimistic case is symmetric.

Both the realistic and the optimistic traders trade with the same intensity on private information ($\beta^{un} = \beta^r$). An optimist adds up a positive part to his market order. Since $\alpha^{un} > 0$ and if he received a positive signal, he increases his market buy order whereas if he received a negative signal he reduces his market sell order. This is proportional to his level of optimism, a.[8] The observed order flow is rationally priced and therefore a price bubble does not occur here. The price and the market depth (the slope of the price function) are equal to the price and market depth predicted by a model a la Kyle (1985) with $M+N$ realistic traders. This can be explained as follows. When pricing the risky asset, the market maker observes a larger positive aggregate order flow or a smaller negative one (inflated towards positive aggregate order flow), called y^{op} as optimists are present. Given the additive and deterministic nature of this extra order flow and given that the market maker rationally anticipates the behavior of the optimists, she can compute the exact size of this extra order flow. In addition, this part is independent of $\tilde{v}$, therefore when trying to extract information from the order flow, the market maker uses a "discounted" order flow, called y^*. This "discounted" order flow is obtained by subtracting from the inflated and actual order flow the extra order flow. Since (i) both types of traders respond identically to private information and (ii) their responsiveness to private information correspond to the one obtained in Kyle (1985), the market depth is also identical to the one predicted by a model a la Kyle (1985). This results in a downward shift of the price function such that the price, given an order flow y^*, is identical to price with an order flow y^{op} as both observed aggregate order flows incorporate the same information. This point is illustrated below in figure 1 as well as the case for which pessimists are present. As this is rationally anticipated by the realistic trader, the misperception of the asset return only affects the optimist's market order, i.e. $\alpha^r = 0$.

[8]This reduction may be so large, if his misperception is very large, that he might submit a market buy order.



The slope for each price function is the same and equal to $\lambda = \frac{\sigma_v\sqrt{(M+N)(1+\tau)}}{2\tau+M+N+1}$.

Figure 1: Overall level of prices for the cases where unrealistic traders are pessimistic, no unrealistic traders are present (Kyle (1985)), and unrealistic traders are optimistic.

We now have a look at some of the comparative statics of the model. Except for the overall effect of σ_ε^2 on the market order from the unrealistic, all the comparative statics concerning the market depth, β^r and β^{un} accord to intuition and are similar to Kyle (1985). We now discuss in more detail the effect of σ_ε^2. Let us rewrite the market order from the unrealistic j as follows

$$x^{un} = \beta^{un}\left(s_j + 2\tau a\right).$$

On the one hand, an increase of the noise in the private signal decreases the size of the market order. On the other hand, it increases the weight within the order due to the misperception of the expectation of prior information. In other words, the noisier the information is, the more unrealistic the unrealistic trader is and the opposite is also true. When $\sigma_\varepsilon^2 = 0$, the unrealistic trades only on private information, however when $\sigma_\varepsilon^2 = +\infty$ the signal is not informative and the unrealistic trader trades only on his misperception a.

We now look at the unconditional expected profit of the traders.

Proposition 2 *The unconditional expected profit for both types of traders are identical. They are equal to*

$$E\left(\Pi^{un}\right) = E\left(\Pi^{r}\right) = \frac{\sigma_v\sigma_u}{2\tau+M+N+1}\sqrt{\frac{\tau+1}{M+N}}.$$

Proof. See Appendix. ■

The wrong beliefs about the mean of the returns of the risky asset have no impact on the level of unconditional expected profit achieved by the unrealistic traders. This is due to the fact that the wrong beliefs have only an impact on the overall level of prices and no impact on the liquidity parameter or on the traders' intensity concerning their private information. Again, the comparative statics are identical to the ones obtained in a model a la Kyle (1985). Both types of traders obtain positive expected profit. In that case, it is not costly to be unrealistic.

3.2 Additive and Multiplicative Misperception

In that subsection, we look at the general case of positive/negative illusions which we call optimism/pessimism. We define it as being made up by two independent parts[9]

- pure optimism/pessimism: misperceptions of the distribution of prior information (expectation and variance),
- pure overconfidence/underconfidence: misperception of the variance of the noise in the private signal.

An optimistic (pessimistic) trader underestimates (overestimates) the variance of both the returns of the asset and the noise in the signal received. An unrealistic trader behaves as if the signal, he received, $\tilde{s}_j = \tilde{v} + \tilde{\varepsilon}_j$, for $j = 1, ..., M$, were drawn according to the following two distributions

$$\begin{aligned} \tilde{v} &\rightarrow N\left(a, \kappa_1 \sigma_v^2\right), \\ \tilde{\varepsilon}_j &\rightarrow N\left(0, \kappa_2 \sigma_\varepsilon^2\right), \end{aligned}$$

where $0 < \kappa_1 < 1$, $0 < \kappa_2 < 1$ for optimistic traders and $\kappa_1 > 1$, $\kappa_2 > 1$ for pessimistic traders. Whenever $\kappa_1 = \kappa_2 = 1$, the unrealistic trader does not misperceive both variances.

We now characterize the equilibrium.[10]

Proposition 3 *Whenever*

$$\begin{aligned} \kappa_1^2 \left(M (1-\tau)(1+2\tau)^2 + N(1+\tau)\right) + 2\kappa_1\kappa_2\tau \left(M(1+2\tau)^2 + 2N(1+\tau)\right) \\ +4\kappa_2^2\tau^2(1+\tau) N \geq 0. \end{aligned} \tag{1}$$

there exists an equilibrium of the following form:

$$\begin{aligned} x_i^r &= \alpha^r + \beta^r s_i, \quad \forall i = 1, ..., N, \\ x_j^{un} &= \alpha^{un} + \beta^{un} s_j, \quad \forall j = 1, ..., M, \end{aligned}$$

$$p = \mu + \lambda y = \mu + \lambda \left(\sum_{i=1}^{N} x_i^r + \sum_{j=1}^{M} x_j^{un} + u \right).$$

The coefficients are such that:
for the optimistic/pessimistic traders

$$\begin{aligned} \alpha^{un} &= \frac{2(1+2\tau)\kappa_2\tau\sigma_u}{\sigma_v\sqrt{M\kappa_1(1+2\tau)^2\left[\kappa_1(1-\tau)+2\kappa_2\tau\right] + N(\kappa_1+2\kappa_2\tau)^2(1+\tau)}} a \\ \beta^{un} &= \frac{\kappa_1(1+2\tau)\sigma_u}{\sigma_v\sqrt{M\kappa_1(1+2\tau)^2\left[\kappa_1(1-\tau)+2\kappa_2\tau\right] + N(\kappa_1+2\kappa_2\tau)^2(1+\tau)}}; \end{aligned}$$

[9] This follows the definition given in Hilton et al. (2004).

[10] We have also solved the model where the two types of unrealistic traders, namely optimistic and pessimistic traders, trade together with the realistic traders and the liquidity traders. The results are qualitatively similar to the results obtained in that section. The proofs are available upon request from the authors.

for the realistic traders

$$\begin{aligned}
\alpha^r &= 0, \\
\beta^r &= \frac{(\kappa_1 + 2\kappa_2\tau)\,\sigma_u}{\sigma_v\sqrt{M\kappa_1\,(1+2\tau)^2\,[\kappa_1\,(1-\tau)+2\kappa_2\tau] + N\,(\kappa_1+2\kappa_2\tau)^2\,(1+\tau)}};
\end{aligned}$$

for the market maker

$$\begin{aligned}
\mu &= -\frac{2\,(1+2\tau)\,\kappa_2\tau}{(2\tau+N+1)\,(\kappa_1+2\kappa_2\tau)+M\kappa_1\,(2\tau+1)}a, \\
\lambda &= \frac{\sigma_v\sqrt{M\kappa_1\,(1+2\tau)^2\,[\kappa_1\,(1-\tau)+2\kappa_2\tau] + N\,(\kappa_1+2\kappa_2\tau)^2\,(1+\tau)}}{\sigma_u\,[(2\tau+N+1)\,(\kappa_1+2\kappa_2\tau)+M\kappa_1\,(2\tau+1)]}.
\end{aligned}$$

Proof. See Appendix. ■

When $\kappa_1 = \kappa_2$, the equilibrium described above is identical to the one found in proposition 1. As long as the unrealistic traders misperceive both variances by the same amount, the effects of the misperceptions onto $E_{un}\,[\tilde{v}|\, s = s_j]$ cancel each other. In all other cases when $\kappa_1 \neq \kappa_2$, both misperceptions affect the variables of the model.

The existence of the equilibrium, for $\kappa_1 \neq \kappa_2$, is subject to condition (1). Whenever that condition is not verified a market breakdown occurs. If unrealistic traders distort their information revelation, by increasing their trading intensity on private information, the equilibrium might fail to exist and a market breakdown might occur. Any misperception leading to a decrease of their trading intensity will not imply a market breakdown. If $\tau < 1$, unrealistic traders have little scope to increase their trading intensity as their private information is already very precise and an equilibrium exists. If $\tau > 1$, unrealistic traders have more scope to increase their trading intensity, however this negative effect is limited if the number of unrealistic traders is low relative to the number of realistic traders ($\frac{M}{N} \leq \frac{1+\tau}{(1-\tau)(1+2\tau)^2}$).[11] In order for the equilibrium to exist when the unrealistic traders are optimistic with a low κ_2 (very overconfident), and, in addition, if $\tau > 1$ and M is large relative to N, the unrealistic traders must be sufficiently "pure optimist" (κ_1 low). Indeed, κ_1 and κ_2 have countervailing effects on β^{un} and therefore being sufficiently "pure optimist" alleviates the impact of being very overconfident. The same thing happens for the case of pessimistic unrealistic traders. In that case being too "pure pessimist" (κ_1 high) might lead to a market breakdown.

[11] If $N = 0$ and provided M and κ_1 are positive, the equilibrium mcondition can be sinplified to $\kappa_1\,(1-\tau) + 2\tau\kappa_2 \geq 0$.

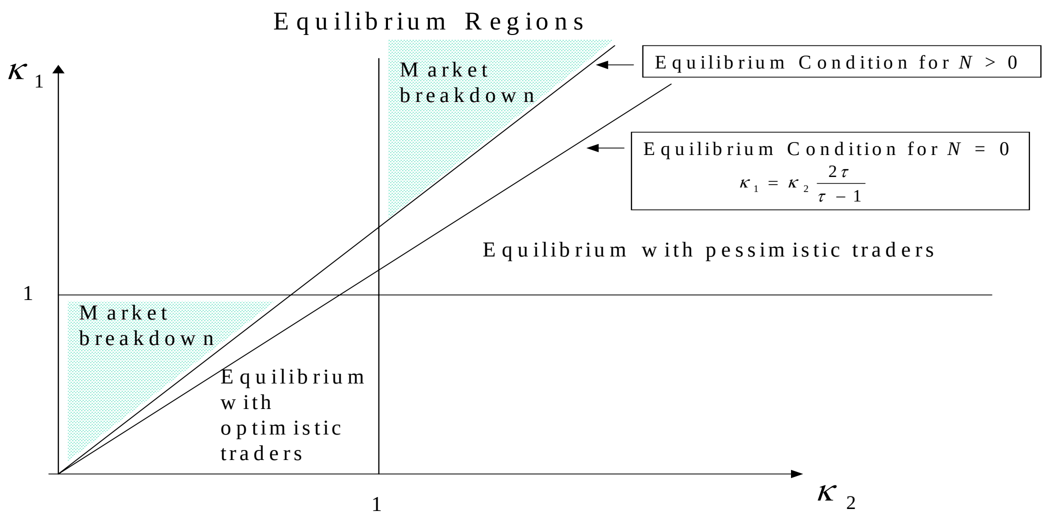


Figure 2: Equilibrium regions, as a function of κ_1 and κ_2, for $N = 0$ whenever $\tau > 1$ and for $N > 0$ whenever $\tau > 1$ and $\frac{M}{N} > \frac{1+\tau}{(\tau-1)(1+2\tau)^2}$.

We now look at the traders' trading behavior. This is done in the following lemma.

Lemma 1 *Comparative Statics on Traders' Behavior*
Provided the equilibrium in Proposition 2 exists, we have

1. β^{un} *increases with* κ_1 *and decreases it with* κ_2,

2. α^{un} *may increase or decrease with* κ_1 *and* κ_2,

3. *when* $(1-\tau)\,\kappa_1 + 2\tau\kappa_2 > 0$, β^r *decreases with* κ_1 *whereas it increases with* κ_1 *if* $(1-\tau)\,\kappa_1 + 2\tau\kappa_2 < 0$; *moreover when* $(1-2\tau)\,\kappa_1 + 2\tau\kappa_2 > 0$, β^r *increases with* κ_2 *and decreases with* κ_2 *if* $(1-2\tau)\,\kappa_1 + 2\tau\kappa_2 < 0$.

Proof. Straightforward by taking the expressions obtained in Proposition 3 and differentiating these expressions by the relevant parameters. ■

When $\kappa_1 > \kappa_2$ ($\kappa_1 < \kappa_2$), the unrealistic trader's responsiveness to private information is higher (lower) than the realistic one.

The unrealistic trading intensity on private information increases with κ_1 and decreases with κ_2. As prior information is misbelieved to be noisier and/or the private information is perceived to be more precise, the unrealistic trades more on private information. The response by the realistic traders to the unrealistic traders' behavior depends on the relative precision of prior information to the noise in private information and on the relative level of error made on the two variances. If prior information is relatively less precise than the noise in private information or if the error made on σ_v^2 is relatively smaller than the one on σ_ε^2, the realistic response is as expected, i.e., he decreases his information revelation with κ_1 and increases it with κ_2 in order to reduce the impact of his market order onto the price. As prior information becomes more precise relative to the noise in the private signal ($\tau > 1$), and κ_1 is large relative to κ_2, the realistic trader's information revelation increase with κ_1 and decrease with κ_2, following the unrealistic's behavior. This is, indeed, the case when $\kappa_1 > \kappa_2 \frac{2\tau}{\tau - 1}$. In that case, the unrealistic trader is trading more intensely on his private information than the realistic trader implying a large impact on price. As a result the realistic trader's marginal impact is smaller allowing him to behave as the unrealistic trader.

The following figures show how κ_1 and κ_2 influence the values of the liquidity parameter, λ, and the parameter μ.

[Insert Figure 3]
[Insert Figure 4]
[Insert Figure 5]
[Insert Figure 6]

The comparative statics regarding μ depend on whether the unrealistic traders are optimistic or pessimistic. For the case where unrealistic traders are optimistic (pessimistic), the parameter μ increases (decreases) with κ_1 whereas it decreases (increases) with κ_2.

The comparative statics on the liquidity parameter, λ, are not affected by whether unrealistic traders are optimistic or pessimistic. We, then, obtain that

- for small κ_2, the liquidity parameter increases and then decreases with κ_1, for large κ_2, the liquidity parameter increases with κ_1,

- for small κ_1, the liquidity parameter decreases with κ_2, for large κ_1, the liquidity parameter increases and then decreases with κ_2.

The following Lemma compares the level of trading intensities obtained in proposition 2, β^{un} and β^{r}, with the one obtained in proposition 1, β^{kyle}, as well as the level of liquidity for proposition 2, λ, with the one of proposition 1, λ^{kyle}.

Lemma 2 *Provided the equilibrium in Proposition 2 exists, we have*

1. *when* $\kappa_1 < \kappa_2$, $\beta^{r} > \beta^{kyle} > \beta^{un}$, *and* $\lambda^{un} > \lambda^{kyle}$,
2. *when* $2\tau(1+\tau)\kappa_2 - \left(2\tau^2 - 1\right)\kappa_1 > 0$ *and* $\kappa_1 > \kappa_2$, $\beta^{un} > \beta^{kyle} > \beta^{r}$, *and* $\lambda^{kyle} > \lambda^{un}$,
3. *when* $2\tau(1+\tau)\kappa_2 - \left(2\tau^2 - 1\right)\kappa_1 < 0$, *we have* $\beta^{un} > \beta^{r} > \beta^{kyle}$, *and* $\lambda^{kyle} \lesseqgtr \lambda^{un}$.

Proof. Straightforward. ■

The first two cases follow intuition. Traders are strategic and take into account the impact of their orders onto the price. Therefore, if the unrealistic traders's trading intensity is low (less than the one of proposition 1), the realistic traders have some scope to increase their trading intensity. This results in a lower liquidity. If the opposite is true, the realistic traders scale down their trading intensity leading to more liquidity in the market. The last case corresponds to the situation where both types of traders trade more intensely than in proposition 1. However, the unrealistic traders' intensity is still larger than the realistic traders' one. Depending on their relative size, the liquidity can be increased or decreased.

We now compute the traders' expected profit.

Proposition 4 *Provided the equilibrium exists, the expected profits are given by for the unrealistic traders*

$$E\left(\Pi^{un}\right) = \frac{\sigma_v^2 \left(1+2\tau\right)^2 \kappa_1 \left[\kappa_1 \left(1-\tau\right) + 2\kappa_2 \tau\right]}{\lambda \left(\left(2\tau + N + 1\right)\left(\kappa_1 + 2\kappa_2 \tau\right) + M\kappa_1 \left(2\tau + 1\right)\right)^2},$$

for the realistic traders

$$E\left(\Pi^{r}\right)=\frac{\sigma_{v}^{2}\left(1+\tau\right)\left[\kappa_{1}+2\kappa_{2}\tau\right]^{2}}{\lambda\left(\left(2\tau+N+1\right)\left(\kappa_{1}+2\kappa_{2}\tau\right)+M\kappa_{1}\left(2\tau+1\right)\right)^{2}}.$$

Whenever $\kappa_1 < \kappa_2$, the realistic traders earn strictly larger expected profit than unrealistic traders.

Provided the equilibrium exists and that $\kappa_2 < \kappa_1$, the unrealistic traders earn

- *negative expected profit* $(\kappa_1(1-\tau)+2\kappa_2\tau<0)$,
- *positive expected profit however lower than the realistic one* $\left(\kappa_2\frac{2\tau(1+\tau)}{2\tau^2-1}<\kappa_1<\kappa_2\frac{2\tau}{\tau-1}\right)$,
- *expected profit larger than the realistic one* $\left(2\tau\left(1+\tau\right)\kappa_2-\left(2\tau^2-1\right)\kappa_1>0\right)$.

Proof. See Appendix. ■

The expected profits are computed under the true distributions of $\tilde{v}$ and $\tilde{\varepsilon}$.

Four different situations arise depending on the value of both τ and the ratio $\frac{M}{N}$ for each case (optimistic or pessimistic): situation 1 is for $\tau\leq\frac{1}{\sqrt{2}}$, situation 2 for $\frac{1}{\sqrt{2}}<\tau\leq1$, situation 3 for $1<\tau$ and $\frac{M}{N}\leq\frac{1+\tau}{(1-\tau)(1+2\tau)^2}$, and situation 4 for $1<\tau$ and $\frac{1+\tau}{(1-\tau)(1+2\tau)^2}<\frac{M}{N}$. We focus on situation 4 as it is the most complete and incorporates the other 3. The graphs for the three other situations are put at the end of the paper (See graphs 9, 10, and 11).

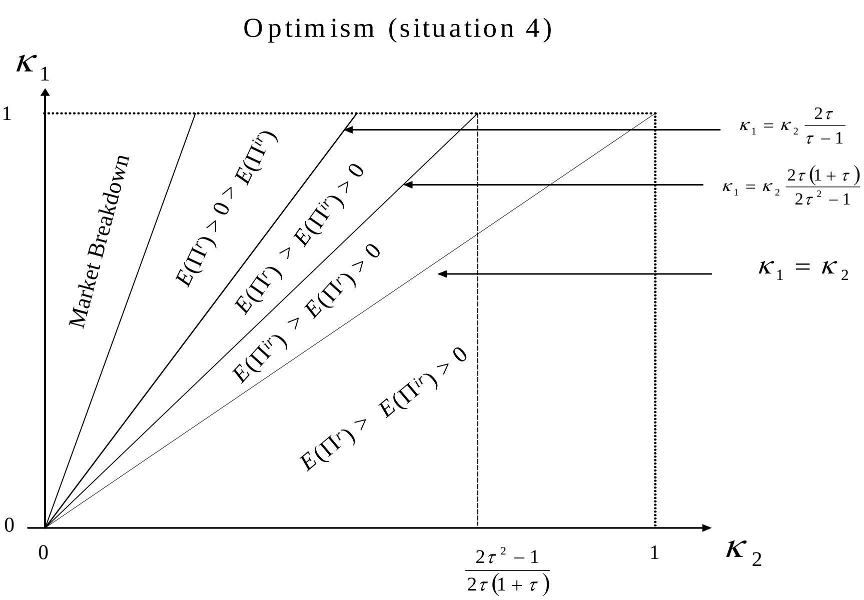


Figure 7: Expected Profit comparison for both a large τ $(1<\tau)$ and a relatively high number of unrealistic traders in the market $(\frac{1+\tau}{(1-\tau)(1+2\tau)^2}<\frac{M}{N})$ with optimistic traders.

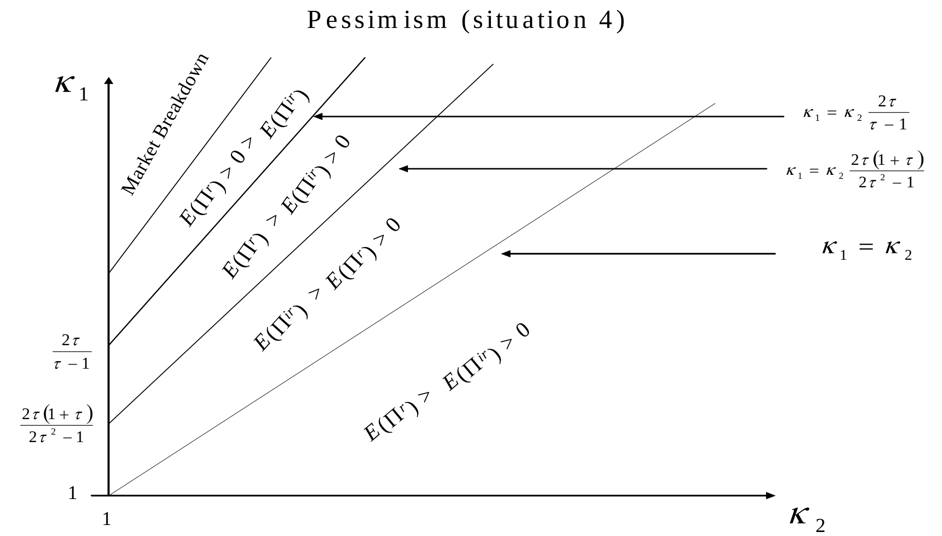


Figure 8: Expected Profit comparison for both a large τ $(1 < \tau)$ and a relatively high number of unrealistic traders in the market $\left(\frac{1+\tau}{(1-\tau)(1+2\tau)^2} < \frac{M}{N}\right)$ with pessimistic.

One can see that whenever $\kappa_2 > \kappa_1$, it is always the case that unrealistic traders earn less expected profit than realistic ones, however they do earn non-negative profit in expected term. In that case, unrealistic traders trade less intensely on their private information than their rational counterpart and the market is less liquid than if all traders were realistic (liquidity in proposition 1).

For $\kappa_1 > \kappa_2$, depending on the relative value of κ_1 with respect to κ_2 and on the value of the other parameters (i.e. depending in which situation of the different ones cited above we are), the realistic traders may earn more expected profit than the unrealistic with the possibility for the latter to earn negative expected profit (situations 3 and 4), or the unrealistic traders may earn on average larger profits than the realistic.

When $2\tau(1+\tau)\kappa_2 - (2\tau^2 - 1)\kappa_1 < 0$, as seen above, both types of traders trade more intensely on their private information than they would if no one were distorting their trading intensity. The effect of the unrealistic is exacerbated by the realistic traders' over-trading. In that case and if both types of traders do not over-trade excessively, the unrealistic traders earn lower expected profit than the realistic traders, however non-negative. When both types do over-trade excessively, the unrealistic traders earn negative expected profit ($\kappa_1(1-\tau) + 2\kappa_2\tau < 0$).

When the effect of unrealistic traders' over-trading is alleviated by the reduction in trading of the realistic traders, the unrealistic traders earn on average more than the realistic traders. This corresponds to the case where $2\tau(1+\tau)\kappa_2 - (2\tau^2 - 1)\kappa_1 > 0$ and $\kappa_1 > \kappa_2$.

For both cases (optimistic and pessimistic traders), as τ increases, the slope of the two lines ($\kappa_2 \frac{2\tau}{\tau-1}$ and $\kappa_2 \frac{2\tau(1+\tau)}{2\tau^2-1}$) becomes flatter implying that the region where the unrealistic trader earns more than the realistic trader shrinks. Ultimately, for an infinite τ and for the parameters where the equilibrium exists, when $\kappa_2 < \kappa_1$, the unrealistic trader earns negative expected profit and the realistic positive, whereas when $\kappa_1 < \kappa_2$ the unrealistic trader earns positive expected profit although lower than the realistic one. When τ is infinite, the unrealistic trader never earns profit, in expected terms, higher than the realistic trader.

We now look more closely at the expected profit functions for both the realistic and the unrealistic.

[Insert Figure 12]
[Insert Figure 13]
[Insert Figure 14]
[Insert Figure 15]

One can see the following comparative statics from the figures above

For the optimist (Figure 12) we obtain that

- for small κ_2, the expected profit increases and then decreases with κ_1, for large κ_2, the expected profit increases with κ_1,

- for small κ_1, the expected profit decreases and then increases with κ_2, for large κ_1, the expected profit increases with κ_2.

For the pessimist (Figure 13) we obtain that

- the impact of κ_1 on the expected profit is the same as the one obtained above for the optimistic,

- for small κ_1, the expected profit decreases with κ_2, whereas for large κ_1, the expected profit decreases and then increases with κ_2.

When optimistic or pessimistic traders are present (Figure 14 and 15) we obtain for the realistic that

- for small κ_2, the expected profit decreases and then increases with κ_1, for large κ_2, the expected profit decreases with κ_1,

- for small κ_1, the expected profit increases with κ_2, whereas for large κ_1, the expected profit decreases and then increases with κ_2.

Figure 12 illustrates the possibility for optimistic traders to obtain negative expected profit. As κ_2 increases, for the optimistic case, the unrealistic trader decreases β^{un}, which reduces his impact on prices and overall increases his profit. For the pessimistic, this is reversed as an increase in κ_2 leads to a more pessimistic trader. The comparative statics are then reversed.

For the realistic trader, the comparative statics do not depend on whether the unrealistic traders are optimistic or pessimistic.

Price efficiency is equal to

$$var\left(v|\, p\right) = \frac{\sigma_v^2 \left(\kappa_1 + 2\tau\kappa_2\right)\left(1+2\tau\right)}{\left(N+2\tau+1\right)\left(\kappa_1+2\tau\kappa_2\right) + M\kappa_1\left(2\tau+1\right)}.$$

We find that price efficiency increases with κ_2 whereas it decreases with κ_1.

The ex-ante volatility is equal to

$$var\left(p\right) = \frac{\sigma_v^2 \left(N\left(\kappa_1 + 2\tau\kappa_2\right) + M\kappa_1\left(2\tau+1\right)\right)}{\left(N+2\tau+1\right)\left(\kappa_1+2\tau\kappa_2\right) + M\kappa_1\left(2\tau+1\right)}.$$

The ex-ante volatility increases with κ_1 and decreases with κ_2.

The effect of κ_1 onto both volatility and price efficiency accords to intuition. However the effect of κ_2 deserves more attention. As κ_2 increases, unrealistic trade less on their private information which leads the realistic to trade more on their private information. The effect on the realistic dominates the effect on the unrealistic leading to an increase in price efficiency. For the effect on the volatility the effect on the unrealistic dominates the effect on the realistic implying a reduction in volatility.

4 Unrealistic Market Makers

We now look at the case where the market maker is unrealistic as well as M traders among the $M+N$ traders. The market maker, given her privileged position in the market, is usually thought to be realistic. However allowing her to be unrealistic enables us to have the same effects as the ones we would obtain in a Grossman and Stiglitz type of model with an unrealistic liquidity supplier.

The unrealistic traders misperceive the distributions of both $\tilde{v}$ and $\tilde{\varepsilon}_j$ as before. Given the fact that the market maker has no access to any private signal, she misperceives the expectation and variance of the distribution of prior information. The market maker believes that the distribution of the asset is such that

$$\tilde{v} \to N\left(\overline{a}, \overline{\kappa}_1 \sigma_v^2\right).$$

As before the market maker behaves competitively.

Proposition 5 *Whenever*

$$\begin{aligned} d_1' = {} & \overline{\kappa}_1 (2\tau+1)(\kappa_1 + 2\kappa_2\tau)(M\kappa_1(2\tau+1) + N(\kappa_1+2\kappa_2\tau)) \\ & -\tau\left(M\kappa_1^2(2\tau+1)^2 + N(\kappa_1+2\kappa_2\tau)^2\right) \geq 0, \end{aligned} \tag{2}$$

there exists a unique linear equilibrium. It is characterized by the following parameters,

for the optimistic/pessimistic traders

$$\begin{aligned} \alpha^{un} &= \frac{2\tau+1}{\sigma_v\sqrt{d_1'}}\left(2\kappa_2\tau a - \overline{a}(\kappa_1+2\kappa_2\tau)\right)\sigma_u, \\ \beta^{un} &= \frac{\kappa_1(2\tau+1)}{\sigma_v\sqrt{d_1'}}\sigma_u, \end{aligned}$$

for the realistic traders

$$\begin{aligned} \alpha^{r} &= -\frac{\overline{a}}{\sigma_v\sqrt{d_1'}}(\kappa_1+2\kappa_2\tau)(2\tau+1)\sigma_u, \\ \beta^{r} &= \frac{\kappa_1+2\kappa_2\tau}{\sigma_v\sqrt{d_1'}}\sigma_u, \end{aligned}$$

for the market maker

$$\begin{aligned} \mu &= \frac{(2\tau+1)}{d}\left[\overline{a}(\kappa_1+2\kappa_2\tau)(M+N+1) - 2M\kappa_2\tau a\right], \\ \lambda &= \frac{\sigma_v}{d\sigma_u}\sqrt{d_1'}, \end{aligned}$$

where $d = M\kappa_1(2\tau+1) + N(\kappa_1+2\kappa_2\tau) + (2\tau+1)(\kappa_1+2\kappa_2\tau)$.

Proof. See Appendix. ■

From the expression of (2), we see that an optimistic market maker exacerbates the market breakdown occurrence whereas a pessimistic one alleviates it. This can be explained as follows. In the following discussion, we only look at the effect of an optimistic market maker as the pessimistic case is symmetric. The market maker's optimism affects the price function in two opposite ways. On the one hand, the market depth decreases with the misperception of the variance, i.e. the higher the misperception is the higher the market depth. An optimistic market maker thinks that the prior information is more precise than it is and therefore believes that the informed private information is less substantial than in reality. As a consequence, she adjusts her price less aggressively. This is done by reducing the liquidity parameter, λ, and therefore by increasing market depth. As a response, informed traders trade more intensely. As they trade more intensely, a market breakdown is more likely to occur. On the other hand, the overall level of price is shifted up due to the misperception of the expectation. Indeed, an optimistic market maker wrongly believes that the expectation of the risky asset is higher than it is and therefore increases the overall level of prices. However that effect is mitigated by the effect of the trader's misperception of the expectation as seen in the equation defining μ. When the situation is symmetric (both the market makers and the unrealistic traders are optimistic or pessimistic), the level of price can either be reduced ($\mu < 0$) or increased ($\mu > 0$). Whenever the situation is asymmetric the shift is positive (negative) with an optimistic (pessimistic) market maker. For any positive shift ($\bar{a} \geq \frac{2M\kappa_2\tau}{(\kappa_1+2\kappa_2\tau)(M+N+1)}a$) or extreme negative shift ($\bar{a} < \frac{2M\kappa_2\tau}{(\kappa_1+2\kappa_2\tau)}a$) of the price function, the unrealistic trader, irrespective of being optimistic or pessimistic, decreases his market order ($\alpha^{un} < 0$) or increases it ($\alpha^{un} > 0$), respectively. This implies that an optimistic trader finding the level of price too high decreases his market order whereas a pessimistic trader finding the level of price too low increases it. For intermediate negative shift of the price ($\frac{2M\kappa_2\tau}{(\kappa_1+2\kappa_2\tau)(M+N+1)}a > \bar{a} > \frac{2M\kappa_2\tau}{(\kappa_1+2\kappa_2\tau)}a$), the unrealistic trader always decreases his market order. This effect induces that both types of informed traders scale down their market order if the market maker is optimistic.

The unconditional expected profits of each traders and for the market maker are given in the following proposition.

Proposition 6 *Provided the equilibrium exists, the expected profits are given by for the unrealistic traders*

$$E\left[\Pi^{un}\right] = \frac{(2\tau+1)^2}{\lambda d^2}\left[\sigma_v^2\kappa_1\left(\kappa_1(1-\tau)+2\kappa_2\tau\right) - \bar{a}\left(\kappa_1+2\kappa_2\tau\right)\left(2\kappa_2\tau a - \left(\kappa_1+2\kappa_2\tau\right)\bar{a}\right)\right],$$

for the realistic traders

$$E\left[\Pi^{r}\right] = \frac{(\kappa_1+2\kappa_2\tau)^2}{\lambda d^2}\left[\sigma_v^2(\tau+1) + \bar{a}^2(2\tau+1)^2\right],$$

for the market maker

$$\begin{aligned} E\left[\Pi^{MM}\right] &= \frac{(2\tau+1)(\kappa_1+2\kappa_2\tau)}{\lambda d^2}\left[\sigma_v^2\left(\bar{\kappa}_1-1\right)\left(M\kappa_1(2\tau+1)+N\left(\kappa_1+2\kappa_2\tau\right)\right)\right. \\ &\quad \left. +\bar{a}(2\tau+1)\left(2M\kappa_2\tau a - \bar{a}\left(\kappa_1+2\kappa_2\tau\right)(M+N)\right)\right]. \end{aligned}$$

Proof. See Appendix. ■

The expected profit of both types of traders is affected by the market marker's misperception on both the expected return of the asset and the variance. The lower the perceived variance (the lower $\bar{\kappa}_1$), the higher the market depth, and therefore the higher the trader's unconditional expected profit. The misperception of the expectation affects differently the two types of traders. The realistic trader's unconditional expected profit increases with it. The analysis for the unrealistic's unconditional expected profit is not as straightforward. Whenever α^{un} and $\bar{a}$ have different sign, the unrealistic trader's expected profit is larger. This happens with an asymmetric situation or may happen with a symmetric one. As before, the unrealistic trader may obtain greater, equal or smaller profit than the realistic one with the possibility for him to have negative expected profit.

Concentrating on the market maker's expected profit

The market maker, when realistic ($\bar{\kappa}_1 = 1$, $\overline{a} = 0$), obtains zero expected profit. However, if she is unrealistic, either optimistic or pessimistic, she may obtain expected profit different from zero. When she is pessimistic (optimistic), an increase (a decrease) of $\bar{\kappa}_1$, increases (decreases) the expected profit through the reduction (increase) in market depth. The effect of the additive misperception is not as straightforward. Indeed, it depends on the sign of the traders' additive misperception. If the situation is asymmetric (market maker is optimistic and traders are pessimistic, or the converse), the second term is always negative. If the situation is symmetric (both pessimistic or both optimistic), the sign of the second term can either be positive or negative.

The ex-ante volatility is equal to

$$var\,(p) = \frac{\sigma_v^2 (N(\kappa_1+2\tau\kappa_2)+M\kappa_1(2\tau+1))((\kappa_1+2\tau\kappa_2)(N+\bar{\kappa}_1(2\tau+1))+M\kappa_1(2\tau+1))}{((N+2\tau+1)(\kappa_1+2\tau\kappa_2)+M\kappa_1(2\tau+1))^2}.$$

It increases with $\bar{\kappa}_1$. As pointed out above an optimistic market maker increases market depth. The informed traders respond to that increase by trading more intensely. When the market maker is pessimistic the opposite is true. In both cases, the effect on the market depth dominates the one on trading intensity leading to the stated comparative static.

The price efficiency is given by

$$var\,(v|\,p) = \frac{\sigma_v^2 \bar{\kappa}_1(\kappa_1+2\tau\kappa_2)(1+2\tau)}{(N+\bar{\kappa}_1(2\tau+1))(\kappa_1+2\tau\kappa_2)+M\kappa_1(2\tau+1)}.$$

The price efficiency increases with $\bar{\kappa}_1$. The price efficiency with an optimistic market is then lower than with a pessimistic market maker. As described before, an optimistic (pessimistic) market maker prices less (more) aggressively by increasing (decreasing) liquidity, traders respond to it by increasing (decreasing) their trading intensity. A higher (lower) trading intensity increases (decreases) information revelation and volatility. The volatility effect always dominates.

5 Conclusion

We develop, here, a model of optimism and pessimism in financial markets. We model unrealistic traders (optimistic/pessimistic) as traders who, as well as misperceiving the expected returns of the asset, can misperceive the variance of both the volatility of the asset returns and the noise in the private signal. An optimistic (pessimistic) trader over-estimate (under-estimate) the expected returns of the asset and can under-estimate (over-estimate) both variances. We study two scenarios, in the first one the unrealistic trader only misperceives the expected returns whereas in the second one, we allow the unrealistic trader to misperceive the expected returns and both variances. In scenario 1, we find that an optimistic (pessimistic) trader purchases (sells) a larger quantity or sells (purchases) a smaller one. We show that the liquidity is not affected by the misperception of the expected returns and is equal to the one we would obtain if all traders were realistic. This is due to the fact that unrealistic traders alter the size of their market order through the misperception of the returns of the asset, without affecting their information revelation. As a consequence, the aggregate order flow faced by the market maker conveys the same amount of information as if all traders were realistic. The expected profit for the unrealistic trader and for the realistic trader are shown to be equal. In scenario 2, we show that a market breakdown can occur. This is indeed the case when both types of traders trade excessively on their private information. If the impact of the unrealistic traders' over-trading is reduced by the realistic traders' trading behavior, the unrealistic traders' expected profit is larger than the realistic one. However, if the impact of that over-trading is not sufficiently reduced by the realistic traders or if the realistic traders trade more intensely on private information than the unrealistic traders, the realistic traders earn on average larger profit than the unrealistic traders. Finally we show that the price efficiency improves with κ_2 whereas it decreases with κ_1, whereas the volatility decreases with κ_2 and increases with κ_1. We also look at the case where the market maker is unrealistic and trade with the unrealistic and realistic traders. First of all, we show that an optimistic market maker exacerbates the occurrence of a market breakdown whereas a pessimistic market maker alleviates it. The introduction of an unrealistic market maker may affect differently the traders. Realistic traders have larger expected profits when the market maker is unrealistic. The implications for the unrealistic traders are not as clear. We, finally, show that when the market maker is unrealistic, her expected profit may be either positive or negative.

An interesting extension of the model would be to look at how the results obtained in the present model would be modified in a dynamic setting. This is left for future research.

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7 Appendix

7.1 Proofs

Proof of Proposition 1 (Equilibrium for Additive misperception) Take the results obtained in proposition 3 for the expression of the parameters α^{un}, β^{un}, α^r, β^r, μ, and λ and set $\kappa_1 = 1$ and $\kappa_2 = 1$.

Proof Proposition 2 (Expected Profits for Additive misperception) Take the results obtained in proposition 4 for the expression of both profits and set $\kappa_1 = 1$ and $\kappa_2 = 1$.

Proof of Proposition 3 (Equilibrium for Additive and Multiplicative misperception)

Given the expressions of the market orders submitted by the optimistic traders, x^o, and by the realistic traders, x^r, the aggregate order flow is equal to

$$y = \sum_{i=1}^{N} x_i^r + \sum_{j=1}^{M} x_j^{un} + u = (N\beta^r + M\beta^{un})\, v + \beta^r \sum_{i=1}^{N} \varepsilon_i + \beta^{un} \sum_{j=1}^{M} \varepsilon_j + N\alpha^r + M\alpha^{un} + u.$$

The unrealistic trader maximizes his conditional expected profit

$$\max_{x_j^{un}} E_{un}\left((v-p)\, x_j^{un} \middle|\, s = s_j \right).$$

Substituting the form of the price as well as the market orders form for the N realistic traders, and the $M-1$ optimistic in the above expression, computing the first order condition and solving it for the market order, we obtain

$$\begin{aligned} x_j^{un} &= \frac{1}{2\lambda}\left[E_{un}\left(v|\, s = s_j \right)\left(1 - (M-1)\,\lambda\beta^{un} - N\lambda\beta^r\right) \right. \\ &\quad \left. -\mu - (M-1)\,\lambda\alpha^o - N\lambda\alpha^r \right]. \end{aligned} \tag{3}$$

We now need to compute $E_{un}\left(v|\, s = s_j \right)$. On one side and given the normality of the random variables we have that $E_{un}\left(v|\, s = s_j \right) = \gamma\left(s_j - E_{un}(v)\right) + E_{un}(v)$ with $\gamma = \frac{cov_{un}(v, s_j)}{var_{un}[s_j]}$. Given that $E_{un}(v) = a$ and $\tau = \frac{\sigma_\varepsilon^2}{\sigma_v^2}$, we obtain

$$E_{un}\left(v|\, s = s_j \right) = \frac{\kappa_1}{\kappa_1 + \kappa_2\tau} s_j + a\frac{\kappa_2\tau}{\kappa_1 + \kappa_2\tau}.$$

Replacing the expression of the conditional expectation into the form of the order (3) and identifying the parameters we have

$$\beta^{un} = \frac{\kappa_1\left(1 - \lambda N\beta^r\right)}{\lambda\left((M+1)\,\kappa_1 + 2\kappa_2\tau\right)}, \tag{4}$$

$$\begin{aligned} \alpha^{un} &= \frac{1}{\lambda(M+1)}\left[a\frac{\kappa_2\tau}{\kappa_1 + \kappa_2\tau}\left(1 - (M-1)\,\lambda\beta^{un} - N\lambda\beta^r\right) \right. \\ &\quad \left. -\mu - \lambda N\alpha^r \right]. \end{aligned} \tag{5}$$

The second order condition is satisfied.

Finally, the realistic maximizes his conditional expected profit. Given his first order condition and the fact that $E\left(v|\, s = s_i \right) = \frac{s_i}{1+\tau}$, the parameters for the realistic's market order are such that

$$\beta^r = \frac{\left(1 - \lambda M\beta^{un}\right)}{\lambda\left(N + 1 + 2\tau\right)}, \tag{6}$$

$$\alpha^r = -\frac{1}{\lambda(N+1)}\left[\mu + \lambda M\alpha^{un}\right]. \tag{7}$$

The second order condition is satisfied.

The market maker behaves competitively and sets a price such that

$$p = E\left[v|\, y\right] = 0 + \frac{cov\left(v, y\right)}{var\left[y\right]}\left(y - E\left(y\right)\right).$$

Given the expression of the aggregate order flow, the parameters of the price schedule are given by

$$\begin{aligned}
\lambda &= \frac{\left(N\beta^{r} + M\beta^{un}\right)}{\left(N\beta^{r} + M\beta^{un}\right)^{2} + \left(N\left(\beta^{r}\right)^{2} + M\left(\beta^{un}\right)^{2}\right)\tau + \frac{\sigma_{u}^{2}}{\sigma_{v}^{2}}}, &(8)\\
\mu &= -\lambda\left(N\alpha^{r} + M\alpha^{un}\right). &(9)
\end{aligned}$$

Solving the above system of six equations defined by equations (4), (5), (6), (7), (8), and (9) for the six unknowns leads to the result of proposition 4.

Proof of Proposition 4 (Expected Profits for Additive and Multiplicative misperception)

The expected profit of any trader, $h = un$ or r, can be written as

$$E\left(\Pi^{h}\right) = E\left(\left(v - p\right)x^{h}\right) = E\left(\left(v - \mu - \lambda y\right)\left(\beta^{h}\left(v + \varepsilon_{h}\right) + \alpha^{h}\right)\right).$$

Given the expression of y and after simplification of some of the terms, the expected profit is equal to

$$E\left(\Pi^{h}\right) = E\left[\left(v - \lambda\left(\left(N\beta^{r} + M\beta^{o}\right)v + \beta^{r}\sum_{i=1}^{N}\varepsilon_{i} + \beta^{un}\sum_{j=1}^{M}\varepsilon_{j} + u\right)\right)\left(\beta^{h}\left(v + \varepsilon_{h}\right) + \alpha^{h}\right)\right].$$

All random variables are independent and have a zero mean, we therefore get

$$E\left(\Pi^{h}\right) = E\left(v^{2}\beta^{h}\left(1 - \lambda\left(N\beta^{r} + M\beta^{un}\right)\right) - \lambda\beta^{h}\varepsilon^{h}\left(\beta^{r}\sum_{i=1}^{N}\varepsilon_{i} + \beta^{un}\sum_{j=1}^{M}\varepsilon_{j}\right)\right).$$

This expression simplifies to

$$\begin{aligned}
E\left(\Pi^{un}\right) &= \beta^{un}\left[\sigma_{v}^{2}\left(1 - \lambda\left(N\beta^{r} + M\beta^{un}\right)\right) - \lambda\beta^{un}\sigma_{\varepsilon}^{2}\right] \text{ for the unrealistic trader,}\\
E\left(\Pi^{h}\right) &= \beta^{r}\left[\sigma_{v}^{2}\left(1 - \lambda\left(N\beta^{r} + M\beta^{un}\right)\right) - \lambda\beta^{r}\sigma_{\varepsilon}^{2}\right] \text{ for the realistic trader.}
\end{aligned}$$

Using the expressions of β^{r}, and β^{un} and after some simplifications we obtain for each type of traders

$$\begin{aligned}
E\left(\Pi^{un}\right) &= \frac{\sigma_{v}^{2}\left(1 + 2\tau\right)^{2}\kappa_{1}\left[\kappa_{1}\left(1 - \tau\right) + 2\kappa_{2}\tau\right]}{\lambda\left(\left(2\tau + N + 1\right)\left(\kappa_{1} + 2\kappa_{2}\tau\right) + M\kappa_{1}\left(2\tau + 1\right)\right)^{2}},\\
E\left(\Pi^{r}\right) &= \frac{\sigma_{v}^{2}\left(1 + \tau\right)\left[\kappa_{1} + 2\kappa_{2}\tau\right]^{2}}{\lambda\left(\left(2\tau + N + 1\right)\left(\kappa_{1} + 2\kappa_{2}\tau\right) + M\kappa_{1}\left(2\tau + 1\right)\right)^{2}}.
\end{aligned}$$

Having the expression of the expected profit for unrealistic traders and for realistic traders, we can compare them to each other. We compute the difference in expected profits, after some rearrangements that leads to

$$E\left(\Pi^{un}\right) - E\left(\Pi^{r}\right) = \frac{\kappa_{1}\left(1 + 2\tau\right)\sigma_{v}^{2}}{\lambda d^{2}}\left(\left(1 + 2\tau\right)^{2}\kappa_{1}\left[\kappa_{1}\left(1 - \tau\right) + 2\kappa_{2}\tau\right] - \left(1 + \tau\right)\left[\kappa_{1} + 2\kappa_{2}\tau\right]^{2}\right),$$

where $d = \left(2\tau + N + 1\right)\left(\kappa_{1} + 2\kappa_{2}\tau\right) + M\kappa_{1}\left(2\tau + 1\right)$.

Given the above expression, finding the sign of $E(\Pi^{un}) - E(\Pi^{r})$ is equivalent to find the sign of

$$(1+2\tau)^2 \kappa_1 \left[\kappa_1 (1-\tau) + 2\kappa_2 \tau\right] - (1+\tau)\left[\kappa_1 + 2\kappa_2 \tau\right]^2 .$$

It is straightforward to prove that the previous expression is equal to

$$2\tau (\kappa_1 - \kappa_2)\left[\kappa_1 \left(1-2\tau^2\right) + 2\kappa_2 (1+\tau)\tau\right]. \tag{10}$$

Whenever $\tau \leq \frac{1}{\sqrt{2}}$, the expression (10) is of the sign of $\kappa_1 - \kappa_2$ and when $\kappa_1 - \kappa_2 > 0$ (< 0), we have $E(\Pi^r) < E(\Pi^{un})$ $(E(\Pi^{un}) < E(\Pi^r))$. Whenever $\frac{1}{\sqrt{2}} < \tau$, (10) has two positive roots $\kappa_1 = \kappa_2$ and $\kappa_1 = \frac{2\kappa_2(1+\tau)\tau}{2\tau^2-1}\kappa_2$. One can prove that the latter is always greater than the former. For any κ_2 and for κ_1 in the interval $\left[\kappa_2, \frac{2\kappa_2(1+\tau)\tau}{2\tau^2-1}\kappa_2\right]$ we have $E(\Pi^r) < E(\Pi^{un})$, for any κ_2 and for κ_1 outside the interval we obtain that $E(\Pi^{un}) < E(\Pi^r)$. Given the expression of the unrealistic's expected profit, one can see that if $\tau > 1$ and $\kappa_1 > \frac{2\tau}{\tau-1}\kappa_2$, unrealistic traders earn negative expected profits.

Proof of Proposition 5 (Unrealistic Market Makers)

After maximizing the traders expected utility we get for the different parameters

$$\begin{aligned}
\alpha^{un} &= -\frac{1}{\lambda(M+1)}\left[\frac{\kappa_{2\tau}}{\kappa_1+\kappa_2\tau}\left(1-\lambda(M-1)\beta^{un}-\lambda N\beta^r\right)\right],\\
\beta^{un} &= \frac{\kappa_1\left(1-\lambda N\beta^r\right)}{\lambda\left((M+1)\kappa_1+2\tau\kappa_2\right)}\\
\alpha^{r} &= -\frac{1}{\lambda(N+1)}\left[\mu+\lambda M\alpha^{un}\right]\\
\beta^{r} &= \frac{1-\lambda M\beta^{un}}{\lambda(N+1+2\tau)}.
\end{aligned} \tag{11}$$

The market maker sets a price, p, such that

$$p = \bar{E}\left[\tilde{v}\middle| y\right] = \bar{E}\left[\tilde{v}\right] + \frac{\overline{cov}(\tilde{v},y)}{\overline{var}(y)}\left(y - \bar{E}(y)\right),$$

where the upper bar denotes that the expectation, covariance and variance are computed given the wrong beliefs of the market maker.

Given the market maker's additive misperception we obtain

$$\lambda = \frac{\left(M\beta^{un}+N\beta^r\right)\overline{\kappa}_1}{\left(M\beta^{un}+N\beta^r\right)^2\overline{\kappa}_1 + \left(M\beta^{un^2}+N\beta^{r^2}\right)\tau + \frac{\sigma_u^2}{\sigma_v^2}}, \tag{12}$$

$$\mu = \left(1-\lambda M\beta^{un}-\lambda N\beta^r\right)\overline{a} - \lambda M\alpha^{un} - \lambda N\alpha^r. \tag{13}$$

Solving the above system of six equations with six unknowns leads to the desired result.

Proof of Proposition 6 (Expected Profit) Follow the same steps as in proposition 4 for the expected profits of the traders.

The market maker's expected profit are equal to

$$E\left[\Pi^{MM}\right] = -NE(\Pi^r) - ME(\Pi^{un}) + E\left(\Pi^{Liq}\right).$$

It is straightforward to show that the expected profit of the liquidity traders, $E\left(\Pi^{Liq}\right)$, are equal to $\lambda\sigma_u^2$. Plug the expressions found for the two types of traders and for the liquidity traders into the expression above and after some manipulations, the desired result is found.

7.2 Figures

The simulations for figures 3, 4, 5, 6, 12, 13, 14, 15, are made for $M = 3$, $N = 10$, $\sigma_v = 1$, $\sigma_u = 1$, $\sigma_\varepsilon = 2$.

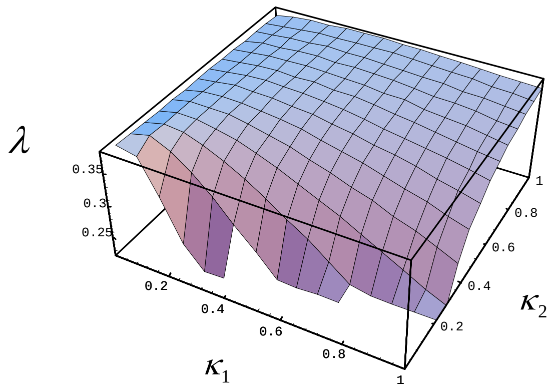

Figure 3: Liquidity parameter as a function of κ_1 and κ_2 whith optimistic traders.

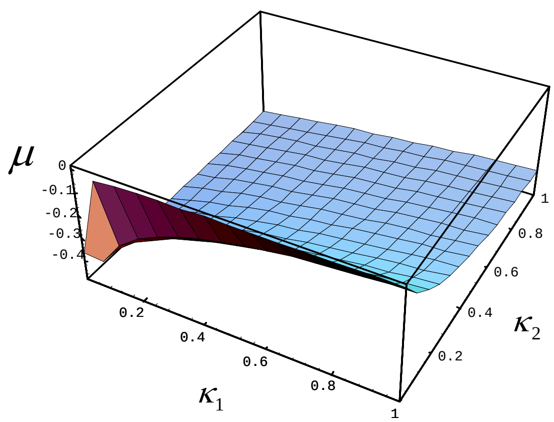

Figure 4: μ parameter as a function of κ_1 and κ_2 whith optimistic traders.

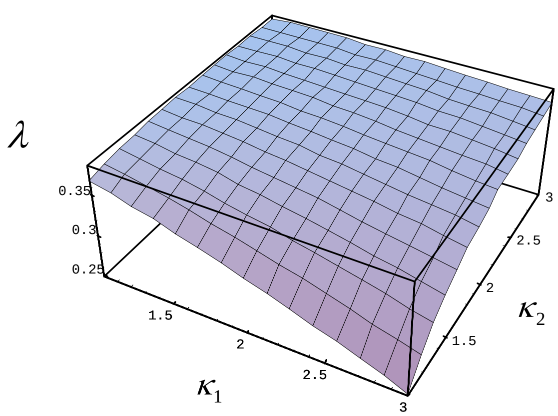

Figure 5: Liquidity parameter as a function of κ_1 and κ_2 whith optimistic traders.

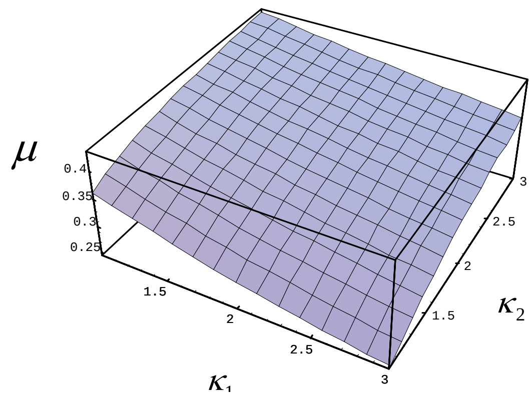

Figure 6: μ parameter as a function of κ_1 and κ_2 whith pessimistic traders.

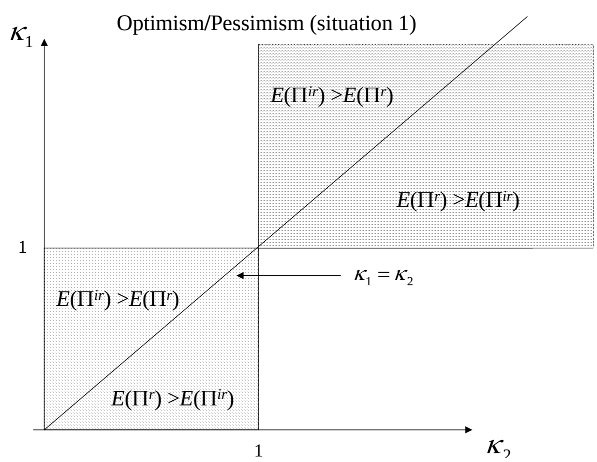

Figure 9: Expected Profit comparison for a low τ ($\tau \leq \frac{1}{\sqrt{2}}$) with optimists/pessimists.

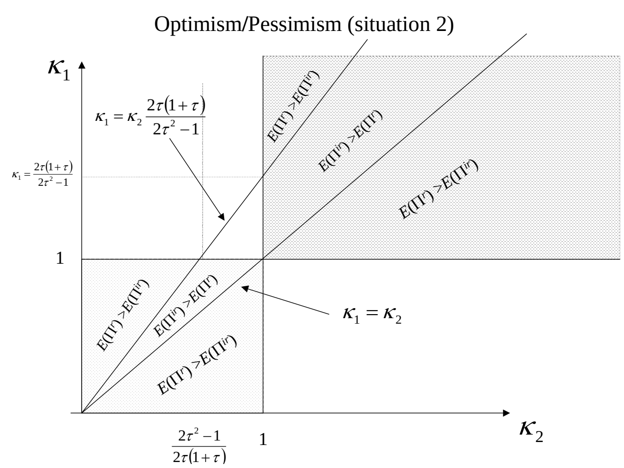

Figure 10: Expected Profit comparison for an intermediate τ ($\frac{1}{\sqrt{2}} < \tau \leq 1$) with optimists/pessimists.

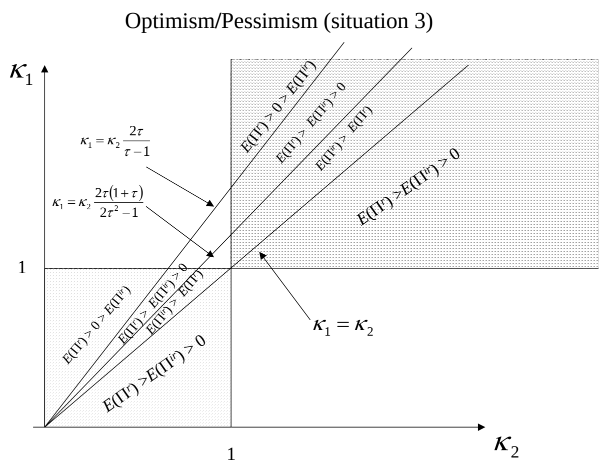

Figure 11**: Expected Profit comparison for both a large** τ $(1 < \tau)$ **and a relatively low number of unrealistic traders in the market** $\left(\frac{M}{N} \leq \frac{1+\tau}{(1-\tau)(1+2\tau)^2}\right)$**, with optimists/pessimists.**

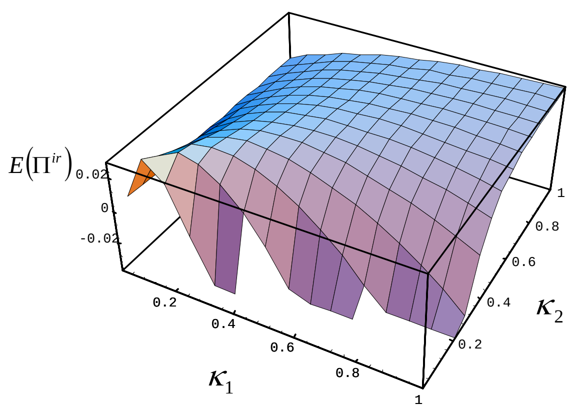

Figure 12**: Expected Profit of the optimistic trader as a function of** κ_1 **and** κ_2**.**

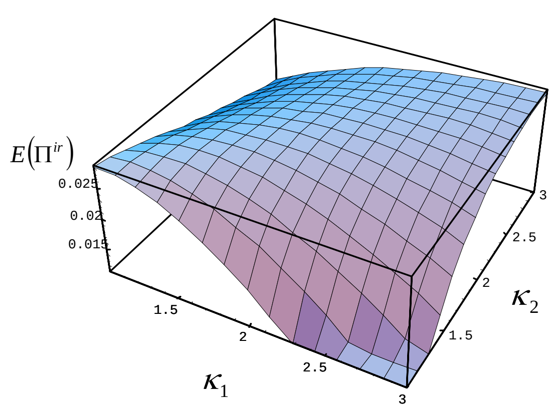

Figure 13**: Expected Profit of the pessimistic trader as a function of** κ_1 **and** κ_2**.**

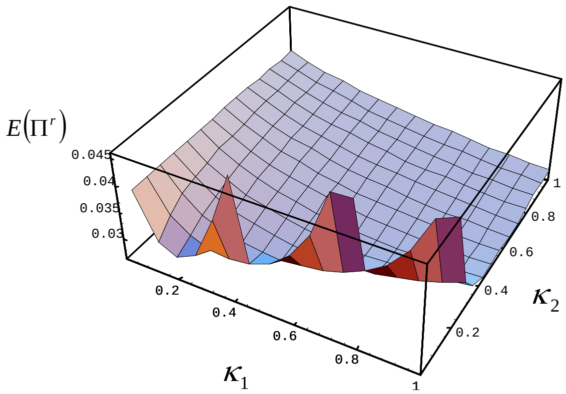

Figure 14: Expected Profit of the realistic tarders as a function of κ_1 and κ_2 when optimistic traders are present in the market.

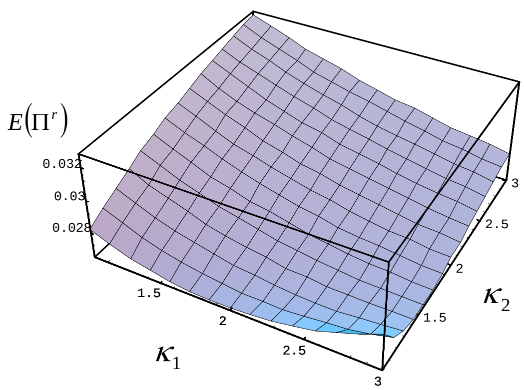

Figure 15: Expected Profit of the realistic traders as a function of κ_1 and κ_2 when pessimistic traders are present in the market.